

Fill in the blanks.

SCORE: _____ / 4 PTS

- [a] The point with rectangular co-ordinates $(-9, -3\sqrt{3})$ has polar co-ordinates $(6\sqrt{3}, \frac{7\pi}{6})$.

NOTE: Both parts of your answer for this question must be positive.

$$r = \sqrt{(-9)^2 + (-3\sqrt{3})^2}$$
$$\tan \theta = \frac{-3\sqrt{3}}{-9} = \frac{\sqrt{3}}{3} \text{ IN } Q_3$$

- [b] The point with polar co-ordinates $(7, -\frac{4\pi}{3})$ has rectangular co-ordinates $(-\frac{7}{2}, \frac{7\sqrt{3}}{2})$.

$$x = 7 \cos(-\frac{4\pi}{3})$$
$$y = 7 \sin(-\frac{4\pi}{3})$$

- [c] The polar co-ordinates $(-6, -\frac{10\pi}{9})$ correspond to the same point as the polar co-ordinates $(6, \frac{17\pi}{9})$ and $(-6, \frac{8\pi}{9})$.

NOTE: Both your answers for this question must be positive.

$$\frac{8\pi}{9} + \pi$$

$$-\frac{10\pi}{9} + 2\pi$$

Convert the polar equation $r = 3 - 5 \cos 2\theta$ to rectangular, and simplify as shown in the website handout. SCORE: ____ / 5 PTS

$$r = 3 - 5(\cos^2\theta - \sin^2\theta) \textcircled{1} \quad \text{OR} \quad r = 3 - 5(1 - 2\sin^2\theta) \textcircled{1}$$

$$r = 3 - 5\left(\frac{x^2}{r^2} - \frac{y^2}{r^2}\right) \textcircled{1}$$

$$r^3 = 3r^2 - 5x^2 + 5y^2 \textcircled{1}$$

$$(x^2 + y^2)^{\frac{3}{2}} = 3(x^2 + y^2) - 5x^2 + 5y^2$$

$$= 8y^2 - 2x^2 \textcircled{1}$$

$$(x^2 + y^2)^3 = (8y^2 - 2x^2)^2 \textcircled{1}$$

$$r = 3 - 5\left(1 - \frac{2y^2}{r^2}\right) \textcircled{1}$$

$$r^3 = 3r^2 - 5r^2 + 10y^2$$

$$= 10y^2 - 2r^2 \textcircled{1}$$

$$(x^2 + y^2)^{\frac{3}{2}} = 10y^2 - 2(x^2 + y^2)$$

$$= 8y^2 - 2x^2 \textcircled{1}$$

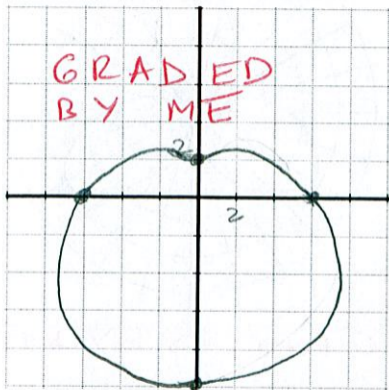
$$(x^2 + y^2)^3 = (8y^2 - 2x^2)^2 \textcircled{1}$$

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ONE
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ONLY

TALK TO ME IF YOU REPLACED $\cos 2\theta$ WITH $2\cos^2\theta - 1$

Consider the graph of the polar equation $r = 6 - 4 \sin \theta$.

SCORE: ___ / 6 PTS



[a] Fill in the blanks.

[i] The shape of the graph is a/an LIMACON WITH DIMPLE ①

[ii] The graph DOES NOT pass through the pole. NO POINTS IF "DIMPLE" MISSING
 does / does not

[iii] Find the rectangular coordinates of the

x - intercept(s)

(6, 0) (-6, 0) ①

y - intercept(s)

(0, 2) (0, -10) ①

θ	r
0	6
$\frac{\pi}{2}$	2
π	6
$\frac{3\pi}{2}$	10

[b] Sketch the graph on the grid provided above. You must provide a scale for the axes & plot all points from part [a][iii] above.

Consider the graph of the polar equation $r = 6 - 6 \sin 3\theta$.

SCORE: ____ / 15 PTS

POLE AXIS POLE

NOTE: $(-r, \theta)$, $(-r, \pi - \theta)$ and $(r, \pi + \theta)$ tests do NOT show that the graph is symmetric

- [a] Using the information above, and the tests and shortcuts shown in lecture, test if the graph is symmetric over the pole, the polar axis, and/or $\theta = \frac{\pi}{2}$. State your conclusions in the table. **NOTE: Run as FEW tests as needed to prove your answers are correct.**

AXIS: $r = 6 - 6 \sin 3(\theta)$ ①
 $r = 6 + 6 \sin 3\theta$ ② X

$\theta = \frac{\pi}{2}$: $r = 6 - 6 \sin 3(\pi - \theta)$ ①
 $r = 6 - 6 \sin (3\pi - 3\theta)$
 $r = 6 - 6 (\sin 3\pi \cos 3\theta - \cos 3\pi \sin 3\theta)$
 $r = 6 - 6 \sin 3\theta$ ① ✓

Type of symmetry	Conclusion
Over the polar axis ②	NO CONCLUSION
Over $\theta = \frac{\pi}{2}$ ②	SYMMETRIC
Over the pole ②	NO CONCLUSION

- [b] Based on the results of part [a], what is the minimum interval of the graph you need to plot first (before using reflections to draw the rest of the graph)?

$[-\frac{\pi}{2}, \frac{\pi}{2}]$ ①

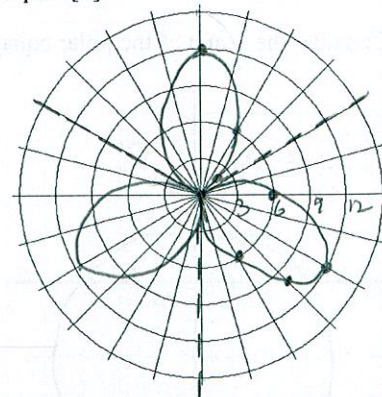
- [c] Find the angles **algebraically** in the minimum interval in part [b] at which the graph goes through the pole.

$0 = 6 - 6 \sin 3\theta$ $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
 ① $\sin 3\theta = 1$ $-\frac{3\pi}{2} \leq 3\theta \leq \frac{3\pi}{2}$
 ① $3\theta = -\frac{3\pi}{2}, \frac{\pi}{2} \rightarrow \theta = -\frac{\pi}{2}, \frac{\pi}{6}$

- [d] Find the value of r (rounded to 1 decimal place) for all common angles in the minimum interval in part [b].

NOTE: You do NOT need to show work, only answers.

① $-\frac{\pi}{4}$	$\frac{\theta}{r}$	0	②
① $-\frac{\pi}{3}$	6	① $\frac{\pi}{6}$	②
① $-\frac{\pi}{4}$	10.2	① $\frac{\pi}{4}$	②
① $-\frac{\pi}{6}$	12	① $\frac{\pi}{3}$	②
① 0	6	① $\frac{\pi}{2}$	②



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- [d] Sketch the graph on the grid provided below. **You must provide a scale for the polar axis & plot all points from part [c] above.**